

A STUDY ON FIXED POINT THEOREM FOR FIVE SELF MAP IN 2- METRIC SPACE

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ABSTRACT

The aim of the present paper is to obtain a fixed point theorem in 2- metric space for continuous densifying mappings by using a generalised inequality.

KEYWORDS: Densifying Mappings, Fixed Point Theorem

1. INTRODUCTION

Bohre & Ram [2] consider 3 mappings A, S & T on a complete 2-metric space (X, d) Such that $AS=SA$, $TA=AT$ and obtained a unique fixed point in X . Jain and Yadav [4] presented a common fixed point theorem for family of self map in 2-metric space.

Here we are interested to find a fixed point theorem in 2-metric space using inequality of continuous densifying mapping

$$(1.1) \quad d(Tx, Ty, a) \leq a_1 \frac{[d(x, Tx, a)d(y, Ty, a)]}{d(x, y, a)} + a_2 \frac{[d(x, T^2x, a)d(y, T^2y, a)]}{d(Tx, Ty, a)} \\ + a_3 [d(x, Tx, a)d(y, Ty, a)] + a_4 [d(x, T^2x, a)d(y, T^2y, a)] + a_5 [d(x, Ty, a) + d(y, Tx, a) + a_6 d(x, y, a)]$$

We are able to obtain some fixed point on the basis of above inequality.

2. MAIN RESULTS

Theorem [2.1] If T be a continuous self map on (X, d) satisfying (1.1) then we get a unique fixed point on (X, d) .

Proof: Let us consider $x_n = T x_{n-1} = T^n x_0$ $n=1, 2, 3, \dots$

Using (1.1) replacing x by x_{n-1} & y by x_n we have

$$(2.1) \quad d(Tx_{n-1}, Tx_n, a) \leq a_1 \frac{[d(x_{n-1}, Tx_{n-1}, a)d(x_n, Tx_n, a)]}{d(x_{n-1}, x_n, a)} + a_2 \frac{[d(x_{n-1}, T^2x_{n-1}, a)d(x_n, T^2x_n, a)]}{d(Tx_{n-1}, Tx_n, a)} \\ + a_3 [d(x_{n-1}, Tx_{n-1}, a)d(x_n, Tx_n, a)] + a_4 [d(x_{n-1}, T^2x_{n-1}, a)d(x_n, T^2x_n, a)] \\ + a_5 [d(x_{n-1}, Tx_n, a) + d(x_n, Tx_{n-1}, a)] + a_6 [d(x_{n-1}, x_n, a)]$$

Using property of metric space, we get

$$(2.2) \quad d(x_n, x_{n+1}, a) \leq a_1 d(x_n, x_{n+1}, a) + a_2 [d(x_{n-1}, x_n, a) + d(x_n, x_{n+1}, a)] \\ + a_3 [d(x_{n-1}, x_n, a) + d(x_n, x_{n+1}, a)]$$

$$+ a_4 [d(x_{n-1}, x_n, a) + 2d(x_n, x_{n+1}, a)]$$

$$+ a_5 [d(x_{n-1}, x_n, a) + d(x_n, x_{n+1}, a)]$$

$$+ a_6 [d(x_{n-1}, x_n, a)]$$

$$(2.3) \quad d(x_n, x_{n+1}, a)(1 - a_1 - a_2 - a_3 - 2a_4 - a_5) \leq d(x_{n-1}, x_n, a)(a_2 + a_3 + a_4 + a_5 + a_6)$$

$$(2.4) \quad d(x_n, x_{n+1}, a) \leq \frac{(a_2 + a_3 + a_4 + a_5 + a_6)}{1 - (a_1 + a_2 + a_3 + 2a_4 + a_5)} d(x_{n-1}, x_n, a)$$

$$\text{Let } K = a_1 + a_2 + a_3 + 2a_4 + a_5$$

(2.5) Thus (2.4) reduced to

$$d(x_n, x_{n+1}, a) \leq K d(x_{n-1}, x_n, a)$$

Replacing n by $n-1$ we get

$$(2.6) \quad d(x_{n-1}, x_n, a) \leq K d(x_{n-2}, x_{n-1}, a)$$

Repeating the process we have

$$(2.7) \quad d(x_n, x_{n+1}, a) \leq K^2 d(x_{n-2}, x_{n-1}, a)$$

Repeating above process again and again we find the equation

$$(2.8) \quad d(x_n, x_{n+1}, a) \leq K^n d(x_0, x_1, a)$$

If $m > n$ then from the property of 2- metric space we can write

$$(2.9) \quad d(x_n, x_m, a) \leq (K^n + K^{n+1} + \dots + K^{m+1}) d(x_0, x_1, a)$$

Or

$$(2.10) \quad d(x_n, x_m, a) \leq \frac{K^n (1 - K^{m+1}) d(x_0, x_1, a)}{(1 - K)}$$

Since $n \rightarrow \infty$ & $m \rightarrow \infty$ and $K < 1$

Then we have that $\{x_n\}$ is a Cauchy Sequence.

Since X is complete therefore there exists a point $u \in X$

Such that $x_n \rightarrow u$

Hence we have $d(u, Tu, a)$

$$(2.11) \quad d(u, Tu, a) \leq d(u, x_n, a) + d(x_n, Tu, a)$$

$$(2.12) \quad d(u, Tu, a) \leq d(u, x_n, a) + d(Tx_n, Tu, a)$$

$$\begin{aligned}
 (2.13) \quad d(u, Tu, a) &\leq d(u, x_{n-1}, a) + a_1 \frac{[d(x_{n-1}, Tx_{n-1}, a) d(u, Tu, a)]}{d(x_{n-1}, u, a)} + a_2 \frac{[d(x_{n-1}, T^2 x_{n-1}, a) d(x_{n-1}, T^2 x_{n-1}, a)]}{d(Tx_{n-1}, Tu, a)} \\
 &+ a_3 [d(x_{n-1}, Tx_{n-1}, a) d(u, Tu, a)] + a_4 [d(x_{n-1}, T^2 x_{n-1}, a) d(u, T^2 x_{n-1}, a)] \\
 &+ a_5 [d(x_{n-1}, Tu, a) + d(u, Tx_{n-1}, a)] + a_6 [d(x_{n-1}, u, a)]
 \end{aligned}$$

As Since $n \rightarrow \infty$ & $x_n \rightarrow u$ we have

$$(2.14) \quad d(u, Tu, a) \leq 0 + a_1 d(u, Tu, a) + a_2 (d(u, Tu, a) + 2(a_3 + a_4 + a_5) d(u, Tu, a))$$

$$(2.15) \quad d(u, Tu, a) (1 - a_1 - a_2 - 2(a_3 + a_4 + a_5)) \leq 0$$

If $0 < a_1 + a_2 + 2(a_3 + a_4 + a_5) \leq 1$

Hence contradiction, therefore, we get

$$Tu = u$$

Thus u is a fixed point of T . Let z a fixed point of so that

$$u \neq z$$

We can write

$$\begin{aligned}
 (2.16) \quad d(u, z, a) &\leq d(Tu, Tz, a) \\
 &\leq a_1 \frac{[d(u, Tu, a) d(z, Tz, a)]}{d(u, z, a)} + a_2 \frac{[d(u, T^2 u, a) d(z, T^2 z, a)]}{d(Tu, Tz, a)} \\
 &+ a_3 [d(u, Tu, a) d(z, Tz, a)] + a_4 [d(u, T^2 u, a) d(z, T^2 z, a)] \\
 &+ a_5 [d(u, Tz, a) + d(z, Tu, a)] + a_6 [d(u, z, a)]
 \end{aligned}$$

Thus finally we find

$$d(u, z, a) \{1 - 2a_5 - a_6\} \leq 0$$

Since we have

$$0 \leq 1 - 2a_5 - a_6 \leq 1$$

Therefore, from the above equation we get

$$d(u, z, a) = 0$$

$$\text{i.e. } u = z$$

Hence u is a unique fixed point.

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